MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

EME4086 - FINITE ELEMENT METHOD

(ME)

8 OCTOBER 2015
9.00 a.m.- 11.00 a.m.
(2 Hours, Open Book)

INSTRUCTION TO STUDENTS

1. This question paper consists of 6 pages with 5 Questions.
2. Questions 1 to 2 are **OPTIONAL**. Attempt only **ONE** out of the **TWO** questions.
3. Questions 3, 4 and 5 are **COMPULSORY**. You MUST attempt these questions.
4. All questions carry equal marks and the distribution of the marks for each question is given.
5. Please write all your answers in the Answer Booklet provided.

OPTIONAL

Question 1:

A load P = 50kN is applied at the centre of a steel bar as shown in **Figure Q1**. The Young's Modulus of steel is 200 GPa. A gap of 1.0 mm exists between the right wall and the bar. The bar is pinned at the left wall. The bar has a cross sectional area of 25 mm^2.

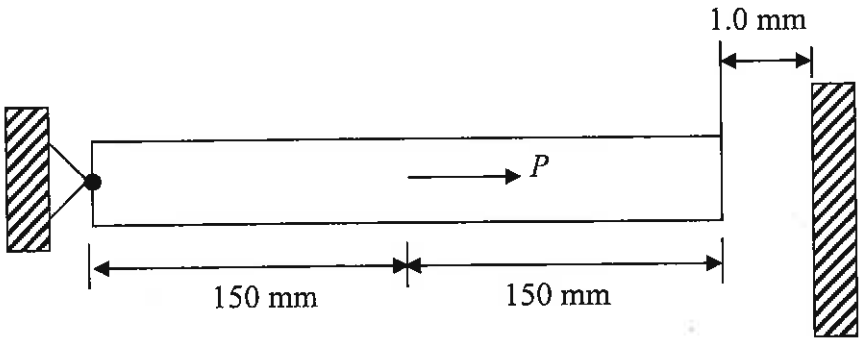

Figure Q1

a) Using the minimum number of finite elements, propose a ONE-DIMENSIONAL linear finite-element model for this problem by:

i) Representing the bar with one-dimensional elements. Please draw the model in your answer script with nodes and elements numbered clearly. Specify the boundary conditions in your sketch as well; **[4 marks]**

ii) Identifying the governing differential equation of the physical problem. **[1 mark]**

b) For the proposal in part **a)**:

i) Calculate the stiffness matrix for each element. **[3 marks]**

ii) Assemble the stiffness matrices into a global stiffness matrix $[K]$. **[3 marks]**

iii) Write down the finite element equation in the form of $[K][U]=[F]$. The matrix $[K]$, and the vectors $[F]$ and $[U]$ should be expressed in numerical values where applicable. **[6 marks]**

iv) Analyze if the gap is closed when the load is applied. **[4 marks]**

v) From your analysis in part **iv)** above, determine the support reactions of the bar. Make a quick analysis to confirm your calculations. **[4 marks]**

Continued...

OPTIONAL

Question 2:

Figure Q2 shows a heating element with one side insulated and the other side attached to a steel plate. The steel plate thickness, L, is 3 cm, and it has a thermal conductivity of 15 W/m-°C. The other face of the plate is exposed to surroundings at an ambient temperature of 25°C, and a convective heat transfer coefficient of 10 W/m^2-°C. Heat is supplied by the heating element at a rate of 300 W/m^2.

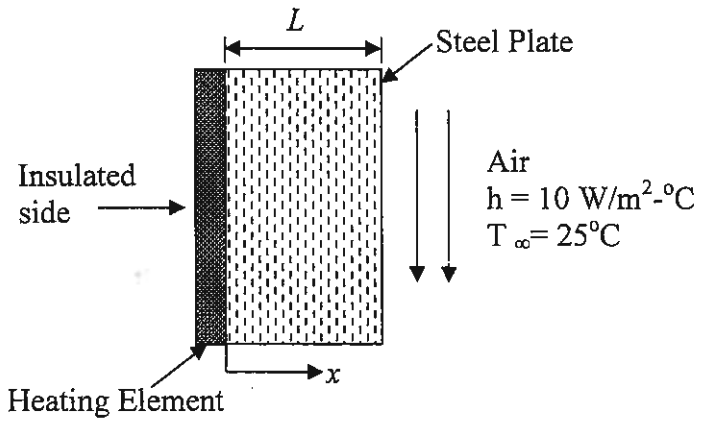

Figure Q2

a) Propose a ONE-DIMENSIONAL linear finite-element model on the domain by:

i) Identifying the governing differential equation of the physical problem; **[1 mark]**

ii) Identifying natural and essential boundary conditions of this problem; **[2 marks]**

iii) Representing the plate with two equal-interval one-dimensional elements, and the convective boundary with a thermal resistor. Please draw the model in your answer script with nodes and elements numbered clearly. **[3 marks]**

b) Determine the heat flux at the exposed face of the plate. **[2 marks]**

c) Calculate the temperature at $x = 0$, $x = L/2$ and $x = L$ by constructing $[K][U]=[F]$. **[11 marks]**

d) Based on governing equations of this problem, justify the use of a linear finite element in part **a)**. **[2 marks]**

e) Without calculation, comment if the temperature at the locations specified in part **b)** and the heat flux in each element will differ if the problem has been modelled using quadratic (3-noded) elements. Explain why. **[4 marks]**

Continued...

COMPULSORY

Question 3:

Consider the linear equation:

$$\frac{du}{dx} - 2u = x^2 \quad \text{for} \quad 0 \le x \le 1 \qquad \text{(Eq. 3.1)}$$

subjected to the boundary condition: $u(0) = -0.25$. Use Galerkin method and assume an initial approximation $u = a + bx + cx^2$, to solve the following:

a) Propose the appropriate function ϕ_0 and trial function ϕ_i such that $u = \phi_0 + \sum_{i=1}^{n} c_i \phi_i$ matches with the approximation of u described above. Explain how ϕ_0 and ϕ_i meet the necessary requirements. **[7 marks]**

b) Determine the numerical values for a, b and c. **[15 marks]**

c) Determine the exact solution for the linear equation and analyze the differences, if any, between the exact solution and the approximate solution in part **b)**. **[3 marks]**

Continued...

COMPULSORY

Question 4:

For the beam shown in **Figure Q4,** the flexural rigidity of the beam is *EI,* and the length of the beam is *2L*. Prior to the application of the uniform load *q*, there is no force in the cable nor is there any slack in the cable. Following the application of the load, the beam deflects such that a tension develops in the cable, and the stiffness of the cable is *k*.

a) Using the minimum number of elements, and in terms of *EI*, *L*, *q*, and *k*,

i) Write the assembled stiffness matrix, the displacement vector and force vector. **[12 marks]**

ii) Express the condensed $[K][U]=[F]$ so that the global displacements at unknown nodes are readily solvable. DO NOT SOLVE for *[U]*. **[7 marks]**

b) Write all necessary equations, without solving them, to demonstrate <u>two ways</u> to determine the reaction at support *A*. Analyze if one way is more accurate than the other. All displacements may be expressed as U_i. **[6 marks]**

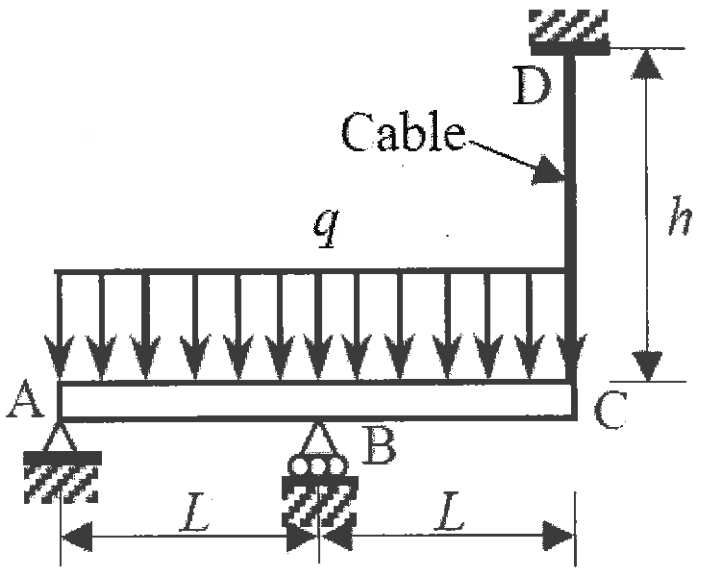

Figure Q4

Continued...

COMPULSORY

Question 5:

A two-dimensional plate with isotropic thermal conductivity, as shown in **Figure Q5**, is subjected to a uniform temperature of 100°C at its top and insulated at the rest of its sides. A uniform heat source of 600 W/m^2 acts over the entire plate.

The plate has been discretized into 8 identical triangles as shown. Elements and global nodes are all labelled, within and without circles, respectively. Your task is to determine temperatures at all nodes by following the steps below.

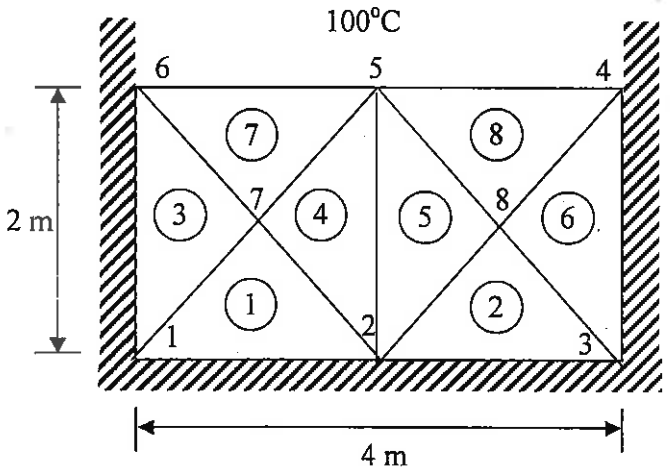

Figure Q5

a) Identify a sub-domain sufficient for your task. You must take advantage of the symmetry of this problem. Draw the sub-domain and specify its boundary conditions clearly in your answer script. **[3 marks]**

b) Given that element (1) with node connectivity *1-2-7* has stiffness matrix [*K*] as follows:

$$[K]=\begin{bmatrix} 5 & 0 & -5 \\ 0 & 5 & -5 \\ -5 & -5 & 10 \end{bmatrix} \text{W/°C}$$

Write the stiffness matrix of an assembled stiffness matrix [*K*] for the sub-domain identified in part **a)**. **[7 marks]**

c) For the sub-domain identified in part **a)**, resolve for temperatures at all nodes by solving the finite element equation [*K*] [*U*] = [*F*]. **[15 marks]**

End of Paper.